Logarithm Scale & dB

Octave & Decade

A logarithmic scale uses decades (factors of 10) or octaves (factors of 2) to represent vast ranges of data, such as frequency, on a compact graph. Decades, useful for spanning magnitudes (10, 100, 1000 Hz), represent a 10:1 ratio, while octaves (doubling) are common in audio.

- **Decade**: Represents a power of 10 increase (e.g., 10 Hz to 100 Hz is one decade; 100 Hz 1000 Hz is a second decade). Often used in electronics, such as Bode plots to display frequency response over a large range.

  There is an increase of one (1) decade from ω1 to ω2 when **ω2/ω1 = 10**

- **Octave**: Represents a doubling of frequency (e.g., 100 Hz to 200 Hz is one octave; 200 Hz to 400 Hz is another). Used in audio and vibration analysis, derived from music.

  There is an increase of one (1)octave from ω1 to ω2 when that **ω2/ω1 = 2**

Power & Voltage gain

It is standard to measure power and voltage gain in decibels:

- **Power** The power gain is: **10 log10 (P2/P1) dB**

- **Voltage** the voltage gain is: **20 log10 (V2/V1) dB**
  Since power is the square of voltage.

From now on, we will drop the base of the logarithm; it is understood to be 10.

Bode & Frequency plots in dB

Bode plots (dB vs. frequency) analyze magnitude and phase Bode Plots analyze the phase vs frequency response. Logarithmic scales are used to display frequency response over several orders of magnitude. The gain is expressed in decibels dB, calculated as 20*log10|a|), and frequency is plotted algorithmically.

Logarithmic plots of a transfer function H(jω), or Bode diagrams of H(jω), are two graphs:

1. A plot of 20 log10 |H(jω)| versus the frequency in log scale (log10 ω)
2. The phase angle of H(jω) versus the frequency in log scale (log10 ω)

The standard representation of the logarithmic magnitude of H(jω) is 20 log10 |H(jω)| dB

Formulas

$$
\begin{aligned}
\log_b(b^x) &= x \\
b^{\log_b x} &= x \\
\log_b 1 &= 0 \\
\log_b b &= 1 \\
\log_b(x \cdot y) &= \log_b x + \log_b y \\
\log_b(\frac{x}{y}) &= \log_b x - \log_b y \\
\log_b(x^y) &= y \cdot \log_b x \\
\log_y x &= \frac{\log_b x}{\log_b y} \quad \text{for any } b > 1
\end{aligned}
$$