Logarithm Scale & dB

Log Scale

A logarithmic scale uses decades (factors of 10) or octaves (factors of 2) to represent vast ranges of data, such as frequency, on a compact graph. Decades, useful for spanning magnitudes (10, 100, 1000 Hz), represent a 10:1 ratio, while octaves (doubling) are common in audio.

- **Decade**: Represents a power of 10 increase (e.g., 10 Hz to 100 Hz is one decade; 100 Hz 1000 Hz is a second decade). Often used in electronics, such as Bode plots to display frequency response over a large range.
- **Octave**: Represents a doubling of frequency (e.g., 100 Hz to 200 Hz is one octave; 200 Hz to 400 Hz is another). Used in audio and vibration analysis, derived from music.

Octave and decade An octave is a frequency band from ω_1 to ω_2 such that $\omega_2 / \omega_1 = 2$ There is an increase in decade from ω_1 to ω_2 when $\omega_2 / \omega_1 = 10$

Motivation for using 20 log10 |H(jω)|

In communications it is standard to measure power gain in decibels,

H

Since power is the square of voltage, the voltage gain is

H

From now on, we will drop the base of the logarithm; it is understood to be 10.

Bode plots dB vs. frequency to analyze magnitude and phase Bode Plots and Frequency Response: Logarithmic scales are used to display frequency response over several orders of magnitude. The gain is expressed in decibels dB, calculated as 20*log10|a|), and frequency is plotted logarithmically.

Logarithmic plots of $H(j\omega)$, or Bode diagrams of $H(j\omega)$, are two graphs: 1. A plot of $20 \log_{10} |H(j\omega)|$ versus the frequency in log scale, that is, versus $\log_{10} \omega$ 2. The phase angle 6 $H(j\omega)$ versus $\log_{10} \omega$ The standard representation of the logarithmic magnitude of $H(j\omega)$ is $20 \log_{10} |H(j\omega)|$ dB

Formulas

$$
\begin{aligned}
\log_b(b^x) &= x \\
b^{\log_b x} &= x \\
\log_b 1 &= 0 \\
\log_b b &= 1 \\
\log_b(x \cdot y) &= \log_b x + \log_b y \\
\log_b(\frac{x}{y}) &= \log_b x - \log_b y \\
\log_b(x^y) &= y \cdot \log_b x \\
\log_y x &= \frac{\log_b x}{\log_b y} \quad \text{for any } b > 1
\end{aligned}
$$